An Efficient Throughput Improvement through Bandwidth Awareness in Cognitive Radio Networks

Tung Thanh Le and Dong-Seong Kim

Abstract: This paper proposes a bandwidth-aware localized-routing algorithm that is capable of sensing the available spectrum bands within a two-hop neighboring for choosing the highly opportunistic routes. A mixed-integer linear programming (MILP) is utilized to formulate the optimization problem. Then, the proposed algorithm is used to determine the maximum bandwidth possible of link pairs via a bandwidth approximation process of relaxed variables. Thereby, the proposed algorithm can allow selected routes corresponding to maximum bandwidth possible between cognitive radio (CR) users through link pairs in cognitive radio networks. By comparing the solution values to previous works, simulation results demonstrate that the proposed algorithm can offer a closed-optimal solution for routing performance in cognitive radio networks. The contribution of this paper is achieved through approximately 50% throughput utilized in the network.

Index Terms: Bandwidth-aware, cognitive radio networks, opportunistic localized-routing, uncertain behavior primary services.

I. INTRODUCTION

Cognitive radio (CR) is a recent and promising development in wireless communications technology [11]–[4]. The tradition of fixed spectrum sharing in licensed communication networks results in inefficient spectrum utilization [5]. Thus, CR is widely considered to resolve the scarcity of spectrum bands and to meet the burgeoning requirements of wireless services [6] by employing opportunistic spectrum sharing, which allows CR users to make efficient use of spectrum bands throughout the network [7]–[10].

One of the key challenges in cognitive radio networks (CRNs) is that how to opportunistically utilize the unoccupied bands in order to effectively exploit them in such networks. In addition, how to select the appropriate routes for assigning resources that can be efficiently utilized since the opportunistic spectrum, that we expect to utilize, varies over time and space in terms of the uncertain behavior of primary services. Therefore, the integration of spectrum awareness and route optimization is the key challenge in facing with effectively spectrum utilization in such networks [11]–[19].

In this paper, we propose a bandwidth-aware opportunistic localized-routing algorithm for CRNs. When the spectrum-aware opportunistic routing is aware of the entire network, it requires a high computation in terms of the exponential variables which correspond to the dynamically network conditions. Therefore, to meet the practical demands, bandwidth awareness based on localized routing is addressed to manage its resource within a two-hop neighboring routing for optimal routes. To this end, we look for the closed-optimal solution in localized routing through the minimization of bandwidth-utilized of link pairs in the network.

The proposed algorithm is based on the bandwidth approximation process (BAP) and the branch-and-bound (B&B) search algorithms. After solving the linear programming (LP) relaxation from the problem formulation in Section IV to determine the lower bound (LB), infeasible solutions have to be sorted in order to reduce the computational time of the CR network. The BAP algorithm filters the approximation solutions (upper bound solutions) that satisfy the condition which is within the vicinity of [LB, (1+\(\varepsilon\))LB]. Hence, the results can be either feasible or infeasible solutions. If a feasible solution is found, it is called a potential optimal solution. Otherwise, infeasible solutions are decomposed into sub-problems through the B&B algorithm to search for a feasible solution. The procedure is iterative until an optimal solution is found. The proposed algorithm can utilize the infeasible solutions that are still significant to be decomposed via B&B algorithm for finding new potential optimal solutions. Then, the proposed algorithm compares among a set of potential optimal solutions to choose the maximum value possible, and it is called an optimal solution. Simulation results show that solutions achieved through the use of the proposed algorithm.

The rest of this paper is organized as follows. Related works...
are discussed about the existing works in Section II. The system model and the problem formulation are issued in Section III and IV, respectively. The proposed algorithm shows how to solve the problem in Section V. Finally, simulation results and the conclusion are given in Section VI and VII, respectively.

II. RELATED WORKS

In this section, we investigate previous works involving multi-hop routing in CRNs. This section is organized as follows. First, we review work on spectrum-aware routing in CRNs. We then review the work on bandwidth-aware routing in such networks. Although bandwidth-aware routing has been studied for multi-hop routing in CRNs [16]–[22], it leads to network routing overheads and as such is not applicable to real networks. To the best of our knowledge, no studies have been conducted on bandwidth-aware localized routing for CRNs to reduce the computational complexity of such networks.

The papers [16] and [20] study the modeling of spectrum sharing and sub-band division, scheduling and interference constraints, and flow routing for multi-hop routing in CRNs. The authors in [20] propose a near-optimal algorithm to solve the mixed-integer nonlinear programming (MINLP) problem for obtaining a feasible solution, but the authors do not consider how to convert a non-linear program to a linear program in their paper. The authors in [16] develop a polynomial-time algorithm to offer highly competitive solutions. They then compare the values to a lower bound obtained from relaxing the MILP problem, to find a solution closer to the optimum.

In other related work, the authors in [21] and [23] propose the same approach with a different model, by considering the joint routing and the frequency scheduling issue in multi-hop CRNs limited by an uncertain spectrum supply. A pair of parameters \((\alpha, \beta)\) is utilized to solve the optimization problem by obtaining a LB and applying the threshold-based coarse-grained fixing algorithm to determine a feasible solution. Thereby, a near-optimal solution to the NP-hard problem is found that minimizes the required network-wide spectrum resource for CR users. In [24], spectrum clouds under multiple cross-layer constraints in multi-hop CRNs are studied through the proposed service provider, called secondary service provider (SSP), to harvest and utilize the available spectrum bands. Through a heuristic relax-and-fix algorithm, feasible solutions can be determined for the optimization problem by relaxing the integer variables. However, the algorithm does not use an iterative approach to find a new feasible solution from infeasible solutions. A new upper bound (UB)\(^1\) for the optimization problem could probably be found if those values are still significant for finding a feasible solution.

In [6], the authors propose an aggregate throughput and robust route set that are determined by rate-based selection strategies, corresponding to links’ throughput, which is maximized. They also propose a polynomial-time algorithm for solving the problem to achieve a near-optimal solution for multi-hop CRNs. However, this paper does not describe how to choose the appropriate routes in the robust route set when considering node interference for multi-hop routing in CRNs.

III. SYSTEM MODEL

To avoid interference between transmission and reception among nodes in the network, all have to listen to their surrounding environment when they want to transmit. Hence, this Section is organized as including the channel-state modeling, interference modeling, and links constraints, for constructing constraints and formula in the following section.

A. Channel-State Modeling

As illustrated in Fig. 2, we can see that nodes A and C can simultaneously send data to nodes F and K, respectively, on the same band \(b\), but in different sub-bands. However, this scene will be interfered by their mutual interference ranges if they had not listened for transmission during the period of time \(T_{ON}\) and \(T_{OFF}\) as illustrated in Fig. 1, thus, among nodes A and C that could probably be interfered by using the same band utilization. In addition, each node in the network uses spectrum sensing techniques to obtain the available spectrum bands through the medium access control layer as discussed in [20] and [25]. In particular, the secondary user senses channels via cooperative sensing and reporting channels [9], and adjust its accessible parameters corresponding to the channel-utilized of primary users. Once the secondary user detects the primary user’s inquiry on its current band in use, the secondary user ceases its transmission for releasing that band in use to the primary user, and start to sense surrounding channels and wait for the next opportunity to transmit [7], [12], [26]–[27]. Therefore, we model the network that each node can listen to available bandwidths before transmission. The outcomes of sensing are binary random sequence for each channel with the periodic sensing in order to obtain the detection quality. For instance, the IEEE 802.22 standard has the small sensing time which is less than 1 ms/channel for fast sensing with energy detection [28]–[29].

Note that the primary users have higher priority over the secondary users. The duration of idle period is the time interval beginning as the channel becomes idle with the last packet sent until the next first packet arrival. The duration of busy period is the time interval beginning at the channel becomes busy with the first packet arrival until the last packet sent. There are sev-

\(^1\) A new upper bound could probably be closer to LB than the previous upper bound.
eral assumptions, including (1) the secondary user chooses one channel corresponding to a sub-channel at one time and (2) the primary users’ arrival process is Poisson process, the arbitration is on the service time distribution with many scenarios such as multimedia traffic, voice traffic [27]. The system can be modeled as a M/G/1 queue2 with multiple inputs. We assume that the duration of busy period of the nth channel is independent and identically distributed (i.i.d), and its idle period distribution function of the nth channel is exponential distribution, and then can be given with the probability density function (pdf) as follows:

\[ f_i(t) = \lambda_i e^{-\lambda_i t} \]  

(1)

where \( t \geq 0; \lambda_i > 0; i = 1, \cdots, N; i \) denotes the duration of the \( i^{th} \) state period time of the \( n^{th} \) channel.

In the scope of this paper, we consider the throughput of link pairs that relies on the unknown behavior of the primary network since the secondary user does not know the definition of the time slot (busy and idle periods) in the primary channel [30]. The OFF state means the available spectrum hole which can be utilized by secondary users (SUs), while the ON state is being occupied by primary users (PUs), as illustrated in Fig. 1. We model two random variables \( T_{ON} \) and \( T_{OFF} \), which are the length of the ON state and OFF state, respectively. Depending on the different types of primary services, \( T_{ON} \) and \( T_{OFF} \) are satisfied different distributions. In this paper, we denote \( f_{ON}(t) \) and \( f_{OFF}(t) \) which can be given as:

\[ T_{ON} \sim f_{ON}(t) = \frac{1}{\lambda_{ON}} e^{-\frac{t}{\lambda_{ON}}}, \]  

(2)

\[ T_{OFF} \sim f_{OFF}(t) = \frac{1}{\lambda_{OFF}} e^{-\frac{t}{\lambda_{OFF}}}. \]  

(3)

According to the expected lengths of the ON and OFF states \( \lambda_{ON} \) and \( \lambda_{OFF} \), these parameters can be estimated by a maximum likelihood estimator [29]. The ON-OFF behavior of the primary service is a renewal process, which is a combination between two Poisson distributions [30]–[31]. Thus, the renewal interval is \( T_R = T_{ON} + T_{OFF} \), and the distribution of \( T_R \), which is denoted by \( f_R(0) \), is given as:

\[ T_{H} \sim f_R(t) = f_{ON}(t) * f_{OFF}(t) \]  

(4)

where “∗” means the convolution operation.

Then we determine the maximum bandwidth possible for opportunistic routing that is described through the maximum link capacity in terms of \( T_{OFF} \) and \( T_{ON} \) of primary services as follows:

\[ C_{ij}^{h_k} = \frac{E[T_{ON}]}{E[T_{ON}] + E[T_{OFF}]} \times C_{ij}^{h_k} = Pb \times C_{ij}^{h_k} \]  

(5)

where \( C_{ij}^{h_k} \) is the maximum bandwidth possible of link pairs depending on \( T_{OFF} \) and \( T_{ON} \); \( E[T_{ON}] \) and \( E[T_{OFF}] \) are the mean expectation of \( T_{ON} \) and \( T_{OFF} \), respectively; \( C_{ij}^{h_k} \) is the available link capacity from node \( i \) to \( j \), which will be defined in (15); and \( Pb \) is the fraction of time which the primary user is busy.

**B. Interference Modeling**

As in Fig. 2, we can see that if node H needs a certain bandwidth for forwarding, it listens to the spectrum sensing information from nodes A, B, C, and K, then determines the minimum bandwidth-utilized on link pairs \{l_{HC}, l_{CK}\}, \{l_{HC}, l_{CB}\}, and \{l_{HC}, l_{CA}\}. We can therefore determine a set of minimum bandwidth between link pairs and then decide the maximum in such a set. Hence, we formulate the optimal capacity for routing at node H as follows:

\[ l_H = \max \{\min\{l_{HC}, l_{CK}\}; \min\{l_{HC}, l_{CB}\}; \min\{l_{HC}, l_{CA}\}\}, \]  

(6)

where \( l_{HC}, l_{CK}, l_{CB}, \) and \( l_{CA} \) are the available capacity on links HC, CK, CB, and CA, respectively. \( l_H \) is the maximum bandwidth possible on the set of minimum bandwidth of link pairs that have a source routing from node H.

Based on the issues mentioned above, we describe an example as follows. In Fig. 3, supposing that node A wants to make a decision for routing to node D. It then has two optional links, link pairs \( (l_{AC}; l_{CD}) \) and \( (l_{AB}; l_{BD}) \). These link pairs have capacities of \( (40; 70) \) and \( (50; 30) \), respectively. First, node A minimizes those link pairs, then it gets the minimum capacity of 40 Mbps for the first link pairs and 30 Mbps for the second one. Then, it maximizes those minimum link pairs, thereby, it can obtain the maximum throughput possible of 40 Mbps. Thus, node A will choose the link pairs \( (l_{AC}; l_{CD}) \) for routing since these link pairs, by avoiding the communication bottleneck, make use of the maximum throughput possible for routing.

Note that (5) and (6) are introduced to show briefly the idea of this paper. While (6) will be obtained by solving problems mentioned in Sections IV and V, (5) is defined to evaluate the maximum bandwidth possible for opportunistic routing based on various behaviors of primary services.
In order to assign sub-bands at a node for transceiver without simultaneously on the same sub-bands, since it will encounter a bottleneck phenomenon in the communication links. Therefore, we can make a constraint as follows:

\[
\sum_{k \in T^h_i} z_{ij}^{hk} + \sum_{p \in T^h_j} z_{jp}^{hk} \leq 1.
\]  

According to constraint (9), if \( z_{ij}^{hk} \) is equal to 1, then \( \sum_{p \in T^h_j} z_{jp}^{hk} \) must be 0, then node \( j \) cannot use sub-band \( k \) for transmission. Otherwise, if \( z_{ij}^{hk} \) is equal to 0, then \( \sum_{p \in T^h_j} z_{jp}^{hk} \leq 1 \), and node \( j \) can transmit to node \( p \) on sub-band \( k \) in band \( h \), but only if node \( p \in T^h_j \).

Scheduling constraints can also be considered. It is clear that if node \( i \) uses sub-band \( k \) in band \( h \) for transmission to node \( j \), then any node that can interfere at node \( j \) will be restricted from using this sub-band. In order to build this constraint, let \( I^h \) be the set of nodes that can interfere at node \( j \) on sub-band \( k \) in band \( h \), giving us:

\[
I^h_j = \{ p : p \neq j, h \in S_p, d_{ip} \leq R^T_j \}. 
\]  

Note that \( R^T \) and \( R^I \) have a mutual relation with the power spectral density (PS) of nodes in the network. When \( PS^T > PS^I \), it means \( R^T < R^I \) as mentioned in [20]. Then, we can formulate:

\[
z_{ij}^{hk} + \sum_{q \in T^h_j} z_{jq}^{hk} \leq 1
\]  

where \( p \in I^h_j \) and \( p \neq i \). If \( z_{ij}^{hk} = 0 \), the interference of the two nodes at node \( j \) but apart from each other can use the same sub-band \( k \) in band \( h \) for their transmission. \(^3\) As illustrated in Fig. 2, when node \( A \) uses sub-band \( k \) in band \( h \) for transmission to node \( B \), other nodes cannot use this sub-band, i.e., nodes \( C, D, E, F \), and \( G \) cannot use it for their transmission. When node \( A \) does not use this sub-band for transmission to node \( B \), all surrounding nodes \( B, C, D, E, F, G \) can use sub-band \( k \) for transmission. In particular, it can be seen that while node \( C \) can use this sub-band for transmission to either node \( H \) or node \( K \), node \( D \) can use it for transmission to either node \( G \) or node \( E \). That means both nodes \( C \) and \( D \) can use the sub-band \( k \) in band \( h \) at the same time without interference. Therefore, Fig. 2 illustrates an example that it adheres to the above constraints (10) and (11).

### C. Links Constraints

When a source node transmits data to a destination node, it may need to relay a number of hops in the intermediate nodes to reach the destination node. However, how to select the appropriate routes for routing that do not exceed the link capacity, is the key point, and therefore, managing the transmission rates in each radio link is needed to prevent the exceeding link capacity. Moreover, when node \( i \) is transmitting to node \( j \) on sub-band \( k \) in band \( h \), their neighboring nodes\(^4\) have to avoid using sub-band \( k \) in band \( h \) for transmission. At the network level, we denote \( l_{ij} \) as the link data rate from node \( i \) to node \( j \), where \( l_{ij} \)

\(^3\)Note that the interference range of a node is twice times of its communication range.

\(^4\)The neighboring nodes are those within the transmission range of nodes \( i \) and/or \( j \).

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>Set of nodes in the network</td>
</tr>
<tr>
<td>( S )</td>
<td>Set of available bands among all nodes in the network</td>
</tr>
<tr>
<td>( S_i )</td>
<td>Set of available bands at node ( i ) in the network</td>
</tr>
<tr>
<td>( W_h )</td>
<td>Bandwidth of band ( h ) in ( S )</td>
</tr>
<tr>
<td>( K_h )</td>
<td>Band ( h ) is divided into sub-bands with unequal bandwidths</td>
</tr>
<tr>
<td>( F_{hk} )</td>
<td>Bandwidth fraction for a sub-band ( k ) in band ( h )</td>
</tr>
<tr>
<td>( R^T_i )</td>
<td>Transmission range of node ( i )</td>
</tr>
<tr>
<td>( R^I_i )</td>
<td>Interference range of node ( i )</td>
</tr>
<tr>
<td>( T^h_i )</td>
<td>Set of available nodes that are using band ( h ) and within the transmission range of node ( i )</td>
</tr>
<tr>
<td>( I^h_j )</td>
<td>Set of nodes which can interfere at node ( j ) on band ( h )</td>
</tr>
<tr>
<td>( PS^T )</td>
<td>Power spectral density of transmission range</td>
</tr>
<tr>
<td>( PS^I )</td>
<td>Power spectral density of interference range</td>
</tr>
<tr>
<td>( z_{ij}^{hk} )</td>
<td>Switching mode that sub-band ( k ) in band ( h ) can either be utilized or not between node ( i ) and ( j )</td>
</tr>
<tr>
<td>( L )</td>
<td>Set of available links in the localized-routing area</td>
</tr>
</tbody>
</table>

From the foregoing, we denote \( S \) as the set of available bands among all nodes in the network and \( S_i \subseteq S \) is the set of available bands of node \( i \in A \). Note that node \( j \in A \) has \( S_j \neq S_i \). In addition, let \( W_h \) be the bandwidth of band \( h \) in \( S \), and band \( h \) can be divided into \( K_h \) sub-bands with unequal bandwidths. In order to assign sub-bands at a node for transceiver without interference between nodes, we suppose that the scheduling of bands and sub-bands must be guaranteed. Hence, assume that band \( h \) can be used in nodes \( i \) and \( j \) if they satisfy the following condition:

\[
z_{ij}^{hk} = \begin{cases} 
1, & \text{if } i \text{ sends data to } j \text{ on sub-band } k \in h; \\
0, & \text{otherwise}.
\end{cases}
\]  

Note that band \( h \in S_{ij} \), where \( S_{ij} = S_i \cap S_j \), which means that band \( h \) is available at node \( i \) and \( j \). Node \( i \in A \) and it uses sub-band \( k \) in band \( h \), within its transmission range, which gives us:

\[
T^h_i = \{ j : j \neq i, h \in S_j, d_{ij} \leq R^T_i \}
\]  

where \( T^h_i \) is the set of nodes that can use the available band \( h \) within the transmission range of node \( i \), \( R^T_i \); \( d_{ij} \) is the distance between node \( i \) and \( j \).

We note that node \( i \) cannot transmit to multiple nodes simultaneously on the same sub-bands, since it will encounter a bottleneck phenomenon in the communication links. Therefore, we
∈ L and belongs to the set of available nodes that are using band h and are within the transmission range of node i, Tih.

Note that if node i is a source node or destination node of link l, the rate of node i is defined as rsrc(l) or rdst(l), respectively. Hence, we have

\[ \sum_{j \in Tih} l_{ij}(l) = r_{src}(l), \quad (12) \]
\[ \sum_{j \in r_{dst}(l)} l_{pj}(l) = r_{dst}(l). \quad (13) \]

Then, we formulate the constraint for two-hop routing which is mentioned as a localized-routing as follows:

\[ \sum_{j \in Tih, j \neq i} l_{ij}(l) = \sum_{p \in Tih} \sum_{j \in r_{src}(l)} l_{ip}(l) \]
\[ + \sum_{j \in r_{dst}(l)} l_{pj}(l). \quad (14) \]

Note that node p in (14) plays a role as an intermediate node in the proposed model.

This model differs from that of [20], where the authors aim to make the complexity of links throughout the entire network. Thus, it is generally impractical in real networks. However, in this paper, we suppose a two-hop neighboring that is applicable to the network, in which condition (14) is satisfied.

In addition, each link data rate cannot exceed the capacity of the link. Therefore, the capacity of link l_{ij} via sub-band k in band h can be described as [32]:

\[ C_{ij}^{hk} = z_{ij}^{hk} \times F_{hk} \times W_{h} \times \log_{2}(1 + \frac{P}{\sigma}) \quad (15) \]

where \( P = g_{ij} \times PS \); \( g_{ij} \) is the power propagation gain; \( PS \) is the power spectral density of a CR node; and \( \sigma \) is the Gaussian noise density. In addition, we assume that all CR nodes have the same \( PS \) for transmission. Note that these parameters have been mentioned in [20], and therefore will not be elaborated in this paper.

From (14) and (15), we have

\[ \sum_{l \in L} \sum_{j \in r_{dst}(l)} \sum_{j \in \{ r_{src}(l) \}} l_{ij}(l) \leq \sum_{h \in S_{ij}} \sum_{k=1}^{K_{h}} C_{ij}^{hk}. \quad (16) \]

IV. PROBLEM FORMULATION

In a multi-hop CR network, the spectrum bands that are available at one node could be utilized by another node in the network. Moreover, a given set of available frequency bands at a particular node that is completely different from the sets of other nodes in the CRN. Hence, the large diversity of the sets of available bands needs to be allocated into sub-bands for utilizing such bands more flexibly in various network conditions.

Mathematically, we formulate the optimization problem based on the minimization of bandwidth-utilized in the network. Thus, we have

\[ \min \sum_{i \in A} \sum_{h \in S_{ij}} \sum_{j \in Tih} \sum_{k=1}^{K_{h}} F_{hk} \times W_{h} \times z_{ij}^{hk}, \quad (17) \]
\[ \text{s.t.} \]
\[ z_{ij}^{hk} + \sum_{q \in Tih} \sum_{j \in r_{src}(l)} \sum_{k \neq i} \sum_{h \in S_{ij}} \sum_{l \in \{ r_{dst}(l) \}} \sum_{k=1}^{K_{h}} \sum_{j \in \{ r_{src}(l) \}} z_{ij}^{hk} \leq 1, \quad (18) \]
\[ \sum_{l \in L} \sum_{j \in r_{dst}(l)} \sum_{j \in \{ r_{src}(l) \}} l_{ij}(l) \leq \sum_{h \in S_{ij}} \sum_{k=1}^{K_{h}} C_{ij}^{hk}. \quad (19) \]

The mathematical formulation of the optimization problem given by (17), (18), and (19) contains binary variables \( z_{ij}^{hk} \). Note that \( F_{hk} \) can be a minimum as 0 and maximum as 1. Therefore, it is possible to linearize the optimization problem as in the mathematical formulation Section from [33] by representing a new set of continuous variables \( D_{ij}^{hk} \in [0, 1] \), which replace the terms \( z_{ij}^{hk} \) in (17). Note that \( D_{ij}^{hk} = z_{ij}^{hk} \times F_{hk} \). Then, variables \( D_{ij}^{hk} \) have to satisfy the following linearization constraints:

\[ D_{ij}^{hk} \leq z_{ij}^{hk}, \quad (20) \]
\[ D_{ij}^{hk} \leq F_{hk}, \quad (21) \]
\[ D_{ij}^{hk} \geq z_{ij}^{hk} + F_{hk} - 1. \quad (22) \]

To sum up, the problem is to minimize in (17), subject to constraints (7), (9), (10), (11), (12), (14), (16), (20), (21), and (22), where \( W_{h}, P, \sigma, r_{src}(l), \) and \( r_{dst}(l) \) are constants, and the optimization variables are \( \eta_{ij}^{hk}, l_{ij}(l) \). Consequently, we have the mixed-integer linear programming (MILP) formulation in terms of an NP-hard problem as follows:

\[ \min \sum_{i \in A} \sum_{h \in S_{ij}} \sum_{j \in Tih} \sum_{k=1}^{K_{h}} W_{h} \times D_{ij}^{hk}, \quad (23) \]
\[ \text{s.t.} \quad (18), (19), (20), (21), \text{and}(22). \quad (24) \]

V. BARCON ALGORITHM

The BARCON algorithm is based on the bandwidth approximation process (BAP) and branch-and-bound (B&B) algorithm. After solving LP relaxation from conditions (23) and (24) in Section IV in order to determine the LB, infeasible solutions need to be sorted to reduce the computational complexity of the network. To this end, the BAP algorithm filters the approximation solutions\(^5\) that satisfy the condition in which they are within the sorting range of the vicinity of \([LB, (1+\varepsilon)\times LB]\) in terms of the LB\(^6\). If a feasible solution is found, it is called a potential optimal solution, and infeasible solutions are continuously decomposed into sub-problems using the B&B algorithm for searching a new feasible solution if infeasible solutions are significant [34]. The procedure iterates until an optimal solution is found.

\(^5\)Approximation solutions are potential optimal solutions as well as potential upper bound solutions.

\(^6\)Note that \( \varepsilon \) is the tolerant accuracy within the range of \( 0 \leq \varepsilon \ll 1 \).
after comparing to maximize the set of solutions as illustrated in Fig. 4.

The operation of the BARCON algorithm is based on the iterative steps as follows:

- **First step**: A LB solution is obtained by solving LP relaxation in polynomial-time. However, the solutions can be infeasible since they are fractional. The BAP algorithm is applied to determine the UB solutions that are potentially optimal solutions.

- **Second step**: The condition of [LB, (1 + ε)LB] is utilized to sort the solutions that do not satisfy the condition. Hence, the set of satisfied solutions are obtained via the condition and then, the minimum sets of such solutions are maximized to select the optimal solution. If no feasible solution is found, the procedure turns to the third step, otherwise, it turns to the fourth step. Note that if infeasible solutions are still significant, these solutions are passed to the third step.

- **Third step**: If there is no feasible solution after the second step, and infeasible solutions are still significant, the B&B algorithm is used to decompose the infeasible solutions into sub-problems for the next iteration loops until an optimal solution is found.

- **Fourth step**: When a set of potential optimal solutions is obtained, those solutions are maximized to find optimal solutions, as described in algorithm 1.

According to the discussion as above, we denote that LB_i and UB_i are the LB and UB of problem i, respectively. In terms of LB_i and UB_i, the minimum LB and UB can be determined as follows.

\[
LB_{\text{min}} = \min_{i \in SP} \{ LB_i \}, \quad (25)
\]

\[
UB_{\text{min}} = \min_{i \in SP} \{ UB_i \}. \quad (26)
\]

where SP is the set of problems. Note that the purpose of (25) and (26) is to shorten the computational time by obtaining (1 + ε) optimal solutions. A problem can be removed from the set of problems if it satisfies

\[
(1 + \varepsilon) \cdot LB_i \geq UB_i. \quad (27)
\]

The current UB solution cannot be removed if the minimum UB solutions are not better than the current optimal solution, as formulated in constraint (27). Otherwise, the current UB solution will be replaced by the minimum UB solution, which is the (1 + ε) optimal solution, as the latest optimal solution as illustrated in Fig. 4.

**Algorithm 1** The BARCON algorithm

1: Initialize the procedure by relaxing all binary variables \( D_{ij}^{hk} \) \( \in [0, 1] \). (This step will relax MILP to LP relaxation).
2: Solve the LP relaxation to determine the LB.
3: With the LB determined by solving the LP relaxation, the BAP is applied to determine the UB with satisfying the condition (LB \( \leq UB \leq (1 + \varepsilon) LB \)).
4: if Solutions obtained satisfy the BAP condition then
5:    Compare to previous potential optimal solutions to select the optimal solutions (maximum bandwidth possible) in the sets.
6:    Step to Line 11.
7: else
8:    Search for finding feasible solutions by B&B search algorithm to decompose infeasible solutions to sub-problems.
9:    Step to Line 2.
10: end if
11: Based on all optimal solutions, solve the optimization problem and establish flows routing to the network.

**VI. SIMULATION PERFORMANCE**

In this section, we describe simulations performed using MATLAB under network scenarios to verify the effectiveness of the proposed algorithm through the contribution of nearly 50% throughput utilized, and thereby improving load-balance in the network. First, we demonstrate bandwidth-aware performance through the efficiency of maximum bandwidth possible on link pairs throughout the network topology. The tolerance accuracy \( \varepsilon \) is evaluated by considering the uncertain behavior of primary users. Therefore, simulation results show that the algorithm can be able to adapt to different scenarios with reliability and scalability in the network.

**A. Bandwidth-aware Performance**

Initially, a network topology is deployed with 100 nodes distributed randomly over the area of 1,000 × 1,000 m². The transmission range of the nodes is 100 meters for CR networks such as, for example, wireless microphones with small transmission ranges as mentioned in [35]. In addition, random bandwidth values are uniformly distributed in the interval of \([0, 35]\) Mbps. The tolerance accuracy \( \varepsilon \) is set at 5%. The bandwidth-utilized by CR users is considered by the busy-idle time \( T_{\text{ON}} \) and \( T_{\text{OFF}} \). In fact, \( T_{\text{ON}} \) and \( T_{\text{OFF}} \) are random variables depending on the primary users [30]. Moreover, \( T_{\text{ON}} \) and \( T_{\text{OFF}} \) are independent and exponential distributions with \( \lambda_{\text{ON}} \) and \( \lambda_{\text{OFF}} \) which are the expected lengths of ON and OFF states corresponding to \( T_{\text{ON}} \) and \( T_{\text{OFF}} \), respectively. Note that \( T_{\text{ON}} \) and \( T_{\text{OFF}} \) are obtained in the ceasing process, we can then evaluate the throughput with the different behaviors of \( T_{\text{ON}} \) and \( T_{\text{OFF}} \) from primary services.

Figs. 6(a), 6(b), and 6(c) illustrate average throughputs corresponding to network topologies Figs. 5(a), 5(b), and 5(c), respectively. Although network topologies have the same size 1,000 × 1,000 m² and number of nodes, the nodes are distributed randomly and the bandwidth-utilized on the network relies on the expected lengths \( \lambda_{\text{ON}} \) and \( \lambda_{\text{OFF}} \). In this paper, we simu-
Fig. 5. Network topology 1000 × 1000 m² with 100 nodes randomly, and different behaviors of primary services in terms of $\lambda_{ON}$ and $\lambda_{OFF}$, respectively: (a) $\lambda_{ON} = 2.6$, $\lambda_{OFF} = 3.6$, (b) $\lambda_{ON} = 1.6$, $\lambda_{OFF} = 2.6$, and (c) $\lambda_{ON} = 3.6$, $\lambda_{OFF} = 4.6$.

Fig. 6. Average throughput corresponding to 100 nodes, with different behaviors of primary services in terms of $\lambda_{ON}$ and $\lambda_{OFF}$, respectively: (a) $\lambda_{ON} = 2.6$, $\lambda_{OFF} = 3.6$, (b) $\lambda_{ON} = 1.6$, $\lambda_{OFF} = 2.6$, and (c) $\lambda_{ON} = 3.6$, $\lambda_{OFF} = 4.6$.

only minimizes the number of link pairs for routing, but also utilizes the maximum bandwidth possible on link pairs in different network scenarios, as can be seen in Figs. 6(a), 6(b), and 6(c), respectively. Therefore, our approach shows that the proposed algorithm can adapt dynamically to network conditions according to $T_{ON}$ and $T_{OFF}$ behaviors in primary services through bandwidth approximation in order to reduce significantly the number of infeasible solutions for routing. Thereby, the network can avoid the hot areas such as traffic congestion.

B. Tolerance Accuracy Evaluation

Tolerance accuracy is intuitively set at 5% in Section VI-A to show the tolerance of UB solutions in sorting range $\epsilon$th. When the tolerance is changed to a higher percentage, simulations show that the BARCON algorithm is still guaranteed to obtain an effective solution in various scenarios where the behavior of primary services is unpredictable.

Through 100 nodes randomly distributed over an area of 1,000 × 1,000 m², the tolerance accuracy $\epsilon$ is adjusted gradually from 8%, to 15%, and 25%, with $\lambda_{ON}$ and $\lambda_{OFF}$ set to 2.6(s) and 3.6(s), respectively. Simulation results obtained in Figs. 7(a), 7(b), 7(c), and 7(d) show that when the tolerance accuracy is adjusted from 5% to 25%, the maximum bandwidth possible in the network still maintains to avoid effectively the traffic congestion. Note that the network topology is set randomly on nodes at each time for evaluating $\epsilon$, so the connectivity could be different from each other. Therefore, the maximum bandwidth possible also changes depending on the connectivity of link pairs in such a network.

VII. CONCLUSION

In this paper, a bandwidth-aware localized-routing algorithm is proposed to choose highly competitive solutions for rout-
ing performance in CRNs. Thereby, the paper’s contribution is achieved nearly 50% throughput utilized as mentioned through simulation results. The optimization problem is determined by using the mixed-integer linear programming. Then, the maximum of the minimization bandwidth possible of link pairs are obtained by using the BARCON algorithm. Simulation results show that the solutions obtained from the proposed algorithm yield a closed-optimal solution for routing performance in CRNs.

As can be seen from the features mentioned above, the BARCON algorithm is completely suitable for applying to large networks since it is capable of reducing the high computational complexity in such networks. The limitation of BARCON is how to enhance the routing performance in the case of multiple overlapping transmissions in the presence of interference throughout the networks.

In future work, we will conduct the optimal routing toward interference-aware opportunistic localized-routing in CRNs that is concerned about the uncertain behavior of primary services in order to improve the routing performance in terms of multiple overlapping transmissions in such networks.

REFERENCES
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